

Exam. Code : 211004  
Subject Code : 4912

M.Sc. (Mathematics) 4<sup>th</sup> Semester (Batch 2020-22)

**TOPOLOGY—II**

**Paper—MATH-582**

Time Allowed—3 Hours] [Maximum Marks—100

**Note** :—Attempt **FIVE** questions in all, selecting at least **ONE** question from each section. The **fifth** question may be attempted from any section. All questions carry equal marks.

**SECTION—A**

1. Give an example of a completely regular spaces which is not normal. Prove that subspaces of completely regular spaces are completely regular and also prove that product of completely regular spaces is completely regular.
2. Let  $A$  be an infinite subset of a regular space  $X$ . Prove that there exists a countable family of open subsets of  $X$ , with mutually disjoint closures such that  $A$  intersects every member of the family.

**SECTION—B**

3. Prove that a subspace of real line is compact if and only if it is closed and bounded. Is the extended real line compact ?

4. Let  $g : X \rightarrow Y$  be a continuous closed surjection such that inverse image of every singleton set is compact (i.e., it has compact fibers). Prove that if  $Y$  is locally compact then  $X$  is also locally compact.

#### SECTION—C

5. Let  $I^X$  be the set of all continuous functions from a space  $X$  to the closed unit interval. Let  $I^X$  have  $n$  members. Let  $P^X$  be the product of  $n$  closed unit intervals  $I$ . Prove that if  $X$  is Tichonov, then  $X$  is homeomorphic to a subspace of  $P^X$ .
6. State and prove Uryshon Metrization Theorem.

#### SECTION—D

7. Give a characterization of an adherent point of a set in terms of nets as well as in terms of Filters.
8. Give an example of a net. Find the filter generated by it. Conversely, give an example of filter and find the net associated with the filter. Show that a net converges to a point if and only if the filter generated by the net converges to that point.